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SS 2019

Algebraic Geometry I

9. Exercise sheet

Exercise 1 (4 Points):

Let S be a scheme over $\mathbb{F}_p$. Recall the absolute Frobenius F_S from Exercise 3 on Sheet 6. Show that F_S is a universal homeomorphism.

Exercise 2 (4 Points):

Let k be a field, and let $f : \mathbb{A}^1_k \to \operatorname{Spec} k$ be the structure morphism. Show that f is closed, but that the base change $f_{\mathbb{A}^1_k} : \mathbb{A}^2_k \to \mathbb{A}^1_k$ is not closed. In particular, f is not universally closed.

Exercise 3 (4 Points):

Let S be a scheme, and let $(X_i), (Y_j)$ be families of S-schemes. Show that the canonical map

$$\left(\coprod_{i,j} X_i \times_S Y_j\right) \to \left(\coprod_i X_i\right) \times_S \left(\coprod_j Y_j\right)$$

is an isomorphism.

Exercise 4 (4 Points):

Let A be a ring, $X = \operatorname{Spec}(A)$ the corresponding affine scheme. Let $x \in |X|$ be a point and let $i : (\{x\}, \mathcal{O}_{X,x}) \to (X, \mathcal{O}_X)$ be the inclusion, considered as a map of locally ringed spaces. Show that if $i_*(\mathcal{O}_{X,x})$ is quasi-coherent (as an $\mathcal{O}_X$-module), then x does not admit a non-trivial generalization.

To be handed in during the lecture on: Wednesday, June 26.